# A Quantum Approach to Optimal Sensor Placement in Self-Driving Cars

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In 2021, BMW launched a quantum computing challenged so solve a set of problems relevant for the automotive life cycle (BMW challenge web-page). The challenge consists of a set of 4 problems concerning design to operation of automobiles. This work addresses the 1st problem. The formulation exploits an Ising Hamiltonian, with the solution relying on quantum quadratic programming. Runtime and complexity analysis is performed, scalability is discussed, which is shown to be polynomial for the problem at hand.

## I. INTRODUCTION

Quantum computing is a promising paradigm which bares great potential in both industry and academia. Quantum algorithms address optimization [1, 2], database search [3], cryptography [4], and satisfiability problems [5]. Recently, it has been gaining momentum with industrial applications, such as traffic flow [6], aircraft load [7], logistics [8], and medical diagnosis [9].

The challenge launched by BMW [10] focuses on 4 problems related to the life cycle of an automobile. The general requirements the challenge are to develop a suitable algorithm implementable on a quantum computer for each problem. Also, the algorithm should be tested on a quantum computer in the cloud, and finally assessed in terms of the performance at scale.

This work considers the 1st BMW problem [11]: the sensor position optimization (SPO) problem. Ensuring robust sensor placement is a critical task when it comes to advance autonomous ground vehicle navigation (GVN). Sensors such as automotive cameras, radar, or lidar, are highly complex systems and hence are large cost factors. While optimizing robust sensor placement promotes save autonomous GVN their use are subjected to certain constraints: Optimal coverage of the vehicle's surroundings, redundant coverage in specific areas, and minimal costs.

Instead of developing a completely new algorithm for future quantum computers, I present a formalization of the SPO problem using Quadratic Programming (QP) [12]. The formulation I consider here covers all requirements of the SPO statement and leads to an IP problem involving binary variables only and having two objective functions to be minimized.

The binary IP (BP) problem is one of Karp's 21 NP-complete problems [13]. Commercial solvers developed to solve IP such as CPLEX [14], FICO [15], and Gurobi [16] fail when problem complexity becomes too high, requiring specialized solvers that are adapted to the specific problem at hand. However, applying any of these solvers to a BP problems such as the one proposed by BMW shows exponential blow-up for relatively small instances.

This project aims to find the optimal and most cost-effective sensor configuration by optimizing the coverage of the region of interest at a minimal price. **First**, a set of sensor candidates will be generated. **Second**, the problem will be mapped to a modification of the exact cover problem. **Finally**, it will be formulated as an Ising Hamiltonian to use with a quantum variational eigensolver.

## II. BMW SENSOR POSITION OPTIMIZATION PROBLEM

The SPO problem is sketched in Figure 1. Following the problem statement, we need to optimize the placement of $N$ sensors, a subset of a maximum number of allowed sensors $K$. Each sensor $S_i$ where $i \in \{0, \ldots, N\}$ is described by a triple $(T_i, P_i, O_i)$ where $T_i$ is the characteristic of the sensor, $P_i$ is the position in cartesian coordinates, and $O_i$ is the orientation in angular coordinates.

The sensor characteristic $T_i$ is a triple $(t_i, fov_i, p_i)$ where $t_i$ describes the numbered type of the sensor, with $0 \rightarrow$ lidar, $1 \rightarrow$ radar, $2 \rightarrow$ camera, and $3 \rightarrow$ ultrasonic. The field of view $fov_i$ is given as vertical and horizontal radial angles, and a range $(\theta, \phi, d)$. Finally, $p_i$ is the price of the sensor, given as a real number.

The problem requires minimizing the cost and maximizing the coverage of certain points of interest such that:

(A) The sensors must be positioned on designated surfaces on the vehicle,

(B) sensors should not be oriented towards the interior of the vehicle,

(C) the number of sensors does not exceed a maximum sensor count of $K$,

(D) all regions of interest are covered by their corresponding sensor types,

(E) points of interest above a certain criticality threshold are covered twice, and

(F) cost of the sensor configuration is minimal.

Constraints (A), (B), and (D) are sketched in Figure 1. For constraint (A), the allowed sensor positions are given as rectangles characterized by their corners; together with the allowed sensor types; Lidar, camera, ultrasound, and/or radar.

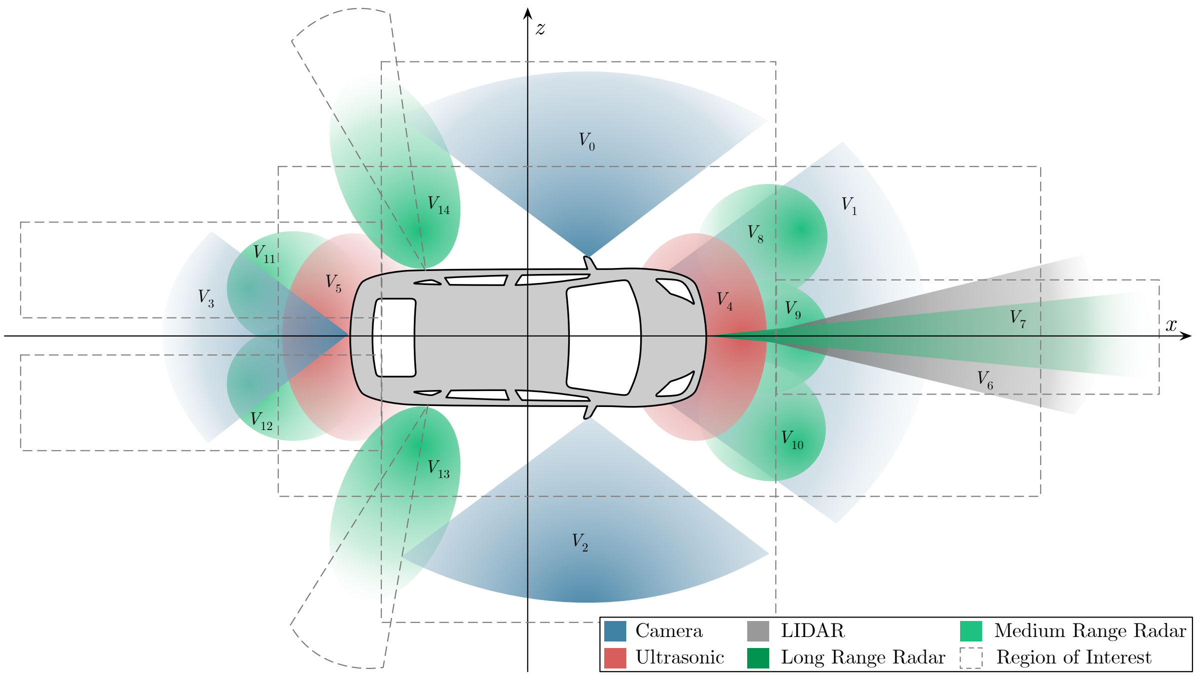


FIG. 1. Simplified sketch of the BMW Sensor Position Optimization problem. The car is to be equipped with sensors of different position, field of view, angle, and type. The total number of sensors is variable. For this SPO, 5 different sensor types are considered, as shown in the legend. The total cost of the sensor configuration should be minimal. All points of interest, marked by cuboids and spherical sectors (dotted lines), must be covered by sensors of a specific type.

Constraint (B) is defined by an occlusion geometry, a polygonal approximation of the shape of the car.

The constraint (C) does not require additional explanation. In (D), the regions of interest are given as cuboids and sectors. The cuboids are assigned criticality parameters, operation type (position, speed, and/or image), and environment type (urban, rural, and/or highway). The sectors, given as spherical segments are assigned the same parameters as the cuboids.

Finally, (E) is given as a critically grid: a discretized sample of the regions of interest, given as cartesian coordinates and a criticality value between 0 and 1.

For a given configuration of sensors, a matrix of region of interest points $R$, and a matrix of Boolean variables $B$ that express coverage, the overall coverage is evaluated as

$$v_{\text{coverage}} = \frac{\sum_{i,j,k} r_{i,j,k} \cdot b_{i,j,k}}{\sum_{i,j,k} r_{i,j,k}} \tag{1}$$

where $i, j, k$ are indices to coordinates in 3D space, $r_{i,j,k}$ is the point of interest at this index, and $b_{i,j,k}$ is the is 1 if the corresponding point is covered and 0 otherwise.

Therefore, the complete optimization problem can be given as:

$$\underset{T,P,O}{\operatorname{argmin}} \left(-w_1 \cdot v_{\text{coverage}} + w_2 \cdot C\right) \tag{2}$$

where $w_1, w_2$ are positive weights and $C = \sum_i p_i$ is overall the cost of the sensor configuration.

## III. QUADRATIC PROGRAMMING FORMULATION

The natural way of mathematically expressing the problem introduced in Section II is using a quadratic program [12].

The problem formulation does not specify the concrete sensors to be considered. Therefore, we begin by assuming certain predefined sensors. We take these from real-world sensor specifications of sensors that are currently on the market, manufactured by Bosch [17] and Livox [18], yielding the base sensors given in Table I.

Next, for the placement of the sensors (A), we identify positions on the surface of the vehicle. For this purpose, we uniformly sample points characterized by the allowed sensor position data. This gives us a set of possible sensor positions.

Third, the orientation of the sensors is identified by sampling angles that cover the half-sphere relative to the plane normal, satisfying constraint (B).

Finally, the sensor specification, position, and angles are used to determine a set of $K$ (satisfying constraint (C)) sensor candidates, which w denoted as $\mathcal{S}$ From this set we wish to take the sensors that are optimal in terms of coverage and cost.

Constraint (D), which reflects sensor coverage, is modeled using set theory: The set of critical points is described by tuples

$$U = \left\{(i, \mathbf{x}_i, c_i) \mid 1 \leq i \leq n, \mathbf{x}_i \in \mathbb{R}^3, c_i \in (0, 1]\right\} \tag{3}$$

| Type | Vertical angle $\theta$ [rad] | Horizontal angle $\phi$ [rad] | Range $d$ [mm] | Price [USD] |
|---|---|---|---|---|
| Lidar | $38.4 \cdot \frac{\pi}{180}$ | $98.4 \cdot \frac{\pi}{180}$ | $90 \cdot 1000$ | 1500.00 |
| Radar (short range) | $80 \cdot \frac{\pi}{180}$ | $80 \cdot \frac{\pi}{180}$ | $30 \cdot 1000$ | 200.00 |
| Radar (long-range) | $6 \cdot \frac{\pi}{180}$ | $3 \cdot \frac{\pi}{180}$ | $210 \cdot 1000$ | 200.00 |
| Camera | $58 \cdot \frac{\pi}{180}$ | $50 \cdot \frac{\pi}{180}$ | $500 \cdot 1000$ | 3000.00 |
| Ultrasonic | $35 \cdot \frac{\pi}{180}$ | $70 \cdot \frac{\pi}{180}$ | $5.5 \cdot 1000$ | 60.00 |

TABLE I. List of base sensors considered for the SPO problem. The sensor specifications (field of view angels and range) and prices have been taken from products by Bosch [17] and Livox [18].

for which $i$ is the identification number, $\mathbf{x}_i$ the cartesian coordinates, and $c_i$ the criticality value.

Adapting [19], it can be assumed without loss of generality that the union of all configurations covers the complete set of critical points, i.e.

$$U = \bigcup_i V_i \tag{4}$$

where $V_i \subset U$ is the set of points covered by sensor $S_i \in \mathcal{S}$.

The target now is to find a subset of the sets $\{V_i\}$, called $R$, such that the pairwise intersection of the sets in $R$ is minimal, with maximal coverage of $U$. Additionally, points in $U$ with a criticality value higher than $\xi = 0.7$ should be covered twice, and the cost for $R$ should be minimal.

## IV. QUANTUM OPTIMIZATION APPROACH

The quantum optimization approach to QP problems is based on adiabatic quantum optimization (AQO) [1] and the concept of a quantum Hamiltonian: we are given a quantum Hamiltonian $H_\mathrm{P}$ whose ground state encodes the solution to a problem of interest, and another Hamiltonian $H_0$, whose ground state is easy to find and to prepare in an experimental setup.

We prepare a quantum system with a ground state of $H_0$. Then, the Hamiltonian for a time $T$ is adiabatically changed according to

$$H(t) = \left(1 - \frac{t}{T}\right) H_0 + \frac{t}{T} H_\mathrm{P}. \tag{5}$$

Then, for a large enough value of $T$, and non-commuting $H_0$ and $H_\mathrm{P}$, the quantum system remains in the ground state for all times according to the adiabatic theorem of quantum mechanics. Measuring the quantum state at time $T$ returns a solution to the problem.

As a first step, we consider only constraint (D) which only requires covering every point of interest exactly once. This constraint is equivalent to the exact cover and can be formulated as follows [19]:

$$H_A = A \sum_{\alpha=1}^{n} \left(1 - \sum_{i:\alpha \in V_i} x_i\right) \tag{6}$$

where $\alpha$ denotes the elements of $U$, with $i$ referring to the subset $V_i$. $H_A = 0$ precisely when every element is included exactly one time, which implies that the unions are disjoint. The existence of a ground state of energy $H = 0$ corresponds to the existence of a solution to the exact cover problem.

Now, to include both constraints (D) and (E), the the Ising Hamiltonian is modified:

$$H_A = A \sum_{\alpha=1}^{n} \left( \left(1 - \sum_{i:\alpha \in V_i} x_i\right)^2 + \left(1 - \sum_{i:\alpha \in V_i \wedge p_i > \xi} x_i\right)^2 \right). \tag{7}$$

The ground state of $H_A = 0$ is reached when every point in $U$ is covered exactly once, except critical points, which are covered exactly twice. If no such state exists, $H_A$ is minimal when the number of superfluous as well as missing points is minimal. It is possible that there are multiple solutions.

Extending this to include the cover with smallest price is done by adding a second energy scale: $H = H_A + H_B$ such that

$$H_B = B \sum_i x_i \cdot p_i. \tag{8}$$

## V. ANALYSIS AND RESULTS

Classically, the sensor placement problem is solved using linear programming [20]. Because this is an NP-complete problem, evaluating each sensor configuration has exponential runtime, with complexity $\mathcal{O}(2^N)$, where $N$ is the number of sensor candidates.

In the quantum domain, a variational quantum eigensolver can be used, needing only $n$ qubits (where $n$ is the number of points of interest), with polynomial runtime.

## VI. CONCLUSION

Classical solutions that optimize sensor placement for self-driving cars compromise an optimal solution for technical feasibility. Due to the speedup provided by quantum computers, it becomes possible to find the truly optimal solution while promising feasible runtimes.

In summary, I have shown how to employ a quantum optimization to tackle the 1st BMW quantum problem efficiently. For solving the 1st BMW problem, which is a sensor placement optimization problem, a QP problem has been formulated that includes all constraints required by BMW. The solution uses an Ising Hamiltonian representation for the problem and solves it using adiabatic quantum optimization. Future work includes moving the problem formulation to real quantum hardware.

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